

# **A HOMOGENIZATION METHODOLOGY FOR THE ANALYSIS OF HIGH FREQUENCY ELASTODYNAMICS IN BOUNDED PERIODIC STRUCTURES**

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Periodic structures appear in nature and in many engineering applications. For example, there are composite materials in which the different material phases are laid out in a periodic fashion. Quite often in structures composed from such materials there are two different length scales: one associated with the dimensions of the overall structure, and another associated with the characteristic size of the constituent materials. If these length scales are significantly disparate then brute force finite element modeling of the entire structure will be computationally expensive, if not impossible.

To overcome this problem a global-local (multiscale) modeling methodology is employed where only a single unit cell is analyzed and the resulting local information is used to homogenize the overall global structure. Homogenization theory has been successfully applied to periodic structures for static analyses, and has recently been applied to wave propagation problems over long time spans [1]. In the current work the goal is to present a homogenization methodology capable of accurately analyzing the dispersive elastodynamics of bounded periodic structures at a wide range of spatial and temporal frequencies. This is an essential requirement for loading scenarios where the wavelength approaches the characteristic length of the constituent materials, as well as for cases involving ballistic forces.

The developed methodology is based on an assumed strain variational method that utilizes multiscale projections, eigenvalue analyses and high-order homogenization theory. An important feature is that the solution field is projected onto a sub-space spanned by a set of vectors that sufficiently characterize the local dynamics of the structure. Numerical results are presented for global-local analyses of layered structures constrained by free or fixed boundaries. This work builds on previously reported results for low frequency [2] and wide frequency range [3] wave motion in infinite periodic structures.

## **References**

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